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QUADRUPLE SERIES EQUATIONS INVOLVING HEAT POLYNOMIALS

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ABSTRACT

In this paper, an exact solution of the quadruple series equations involving heat polynomials $P_{n,\nu}(x,t)$ is given. Certain integral and series representations for $P_{n,\nu}(x,t)$ and $w_{n,\nu}(x,t)$ are shown, which are needed in the course of analysis.

Introduction

In this paper, we consider the following quadruple series equations:

$$\sum_{n=0}^{\infty} A_n \frac{t^{-n}\rho^n}{\Gamma\left(\nu+\frac{1}{2}+n+p\right)} P_{n+p,\nu}(x,-t) = \phi_1(x,t); 0 \le x < y \qquad \textbf{(1.1)}$$

$$\sum_{n=0}^{\infty} A_n \frac{1}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)} P_{n+p,\sigma}(x,-t) = \phi_2(x,t); y < x < z \qquad \textbf{(1.2)}$$

$$\sum_{n=0}^{\infty} A_n \frac{t^{-n}\rho^n}{\Gamma\left(\nu+\frac{1}{2}+n+p\right)} P_{n+p,\nu}(x,-t) = \phi_3(x,t); z < x < h \qquad \textbf{(1.3)}$$

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$$\sum_{n=0}^{\infty} A_n \frac{1}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)} P_{n+p,\sigma}\left(x,-t\right)=\phi_4\left(x,t\right); h<x<\infty \quad \textbf{(1.4)}$$

Where, $\phi_1(x,t), \phi_2(x,t), \phi_3(x,t)$ **and** $\phi_4(x,t)$ **are prescribed functions for t > ρ > 0 and** A_n **is to be determined, and** $P_{n,\nu}(x,t)$ **is the heat polynomial [Haimo, 1966] defined by**

$$P_{n,\nu}\left(x,t\right)=\sum_{k=0}^{n} 2^{2k}\binom{n}{k}\frac{\Gamma\left(\nu+\frac{1}{2}+n\right)}{\Gamma\left(\nu+\frac{1}{2}+n-k\right)} \cdot x^{2n-2k}, t^{k}, \nu>0 \quad \textbf{(1.5)}$$

It may be noted that $P_{n,o}(x,t)=\nu_{2n}(x,t)$ **is the ordinary heat polynomial of even order defined by Rosenbloom and Widder (1959) and that**

$P_{n,o}\left(x,-1\right)=\left(-1\right)^{n} \cdot 2^{2n} n! \cdot L_n^{-\frac{1}{2}}\left(\frac{x^2}{4}\right)=H_{2n}\left(\frac{x}{2}\right)$ **, the Hermite polynomial of even order**

defined by Erdelyi (1853). The analysis given here is purely formal and no attempt is made to supply details of rigorous proof.

2. SOLUTION

Multiplying equation (1.1) by $t^{-\left(p+m+\mu+\frac{1}{2}\right)} \cdot \left(t-\alpha\right)^{\mu+m-\nu-1}$ **where m is a positive integer, integrating with respect to t from ρ to ∞ and using (2.11) we get**

$$\sum_{n=0}^{\infty} \frac{A_n}{\Gamma\left(\mu+m+n+p+\frac{1}{2}\right)} P_{n+p,\mu+m}\left(\xi,-\rho\right)=F_1\left(\xi,\rho\right), 0<\xi<y \quad \textbf{(3.1)}$$

Where $\mu+m>\nu-\frac{1}{2}$ **and**

$$F_1(\xi,\rho)=\frac{\rho^{\nu+\frac{1}{2}}}{\Gamma(\mu+m-\nu)}\int_\rho^\infty t^{-\left(\rho+m+\mu+\frac{1}{2}\right)}\,(t-\rho)^{\mu+m-\nu-1}\phi_1(\xi,t)dt \tag{3.2}$$

Further setting t = ρ in (1.2), multiplying it by $x\left(x^2-\xi^2\right)^{\sigma-\mu-1} e^{-\frac{x^2}{4}}$, integrating with respect to x from ξ to ∞ and applying (2.13) we obtain

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\left(\mu+n+p+\frac{1}{2}\right)}P_{n+p,\mu}(\xi,-\rho)=F_2(\xi,\rho),\, y<\xi<z \tag{3.3}$$

Where $\sigma>\mu>-\frac{1}{2}$ **and**

$$F_2(\xi,\rho)=2^{1-2(\sigma-\mu)}\, e^{-(\sigma-\mu)}\, \mathbf{e}^{\left(\frac{\xi^2}{4\rho}\right)}\int_\xi^\infty x\left(x^2-\xi^2\right)^{\sigma-\mu-1} e^{-\left(\frac{x^2}{4\rho}\right)}\phi_2(x,\rho)dx \tag{3.4}$$

Now multiplying (3.1) by $\xi^{2(\mu+m)-1}$ and applying the operator $\left(\frac{d^2}{d\xi^2}\right)^m$, we see in view of (2.6) that

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)}P_{n+p,\mu}(\xi,-\rho)=H(\xi,\rho),0<\xi<y \tag{3.5}$$

Where $H_1(\xi,\rho)=\xi^{1-2\mu}\left(\frac{d^2}{d\xi^2}\right)^m\left[\xi^{2(\mu+m)-1}F_1(\xi,\rho)\right]$ **(3.6)**

Now, we multiply equation (1.3) by $t^{-\left(p+m+\mu+\frac{1}{2}\right)}(t-\alpha)^{\mu+m-\nu-1}$, where m is a positive integer, integrating with respect to t form ρ to ∞ and using (2.11), we get

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\left(\mu+m+n+p+\frac{1}{2}\right)}P_{n+p+\mu+m}(\xi,-\rho)=F_3(\xi,\rho),z<\xi<h \tag{3.7}$$

Where, $\mu+m>\nu>-\frac{1}{2}$ **and**

$$F_3(\xi,\rho)=\frac{\rho^{\nu+\frac{1}{2}}}{\Gamma(\mu+m-\nu)}\int_\rho^\infty t^{-\left(p+m+\mu+\frac{1}{2}\right)}\,(t-\rho)^{\mu+m-\nu-1}\phi_3(\xi,t)dt \tag{3.8}$$

Setting t = ρ in (1.4), multiplying it by $x\left(x^2-\xi^2\right)^{\sigma-\mu-1} e^{-\frac{x^2}{4}}$, integrating with

respect to x from ξ to ∞ and applying (2.13) we obtain.

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\left(\mu+n+p+\frac{1}{2}\right)}P_{n+p,\mu}\left(\xi,-\rho\right)=F_4\left(\xi,-\rho\right),h<\xi<\infty \tag{3.9}$$

Where $\sigma > m > -\frac{1}{2}$ **and**

$$F_4\left(\xi,\rho\right)=2^{1-2(\sigma-\mu)}\,\rho^{-(\sigma-\mu)}\,\frac{e^{\left(\frac{\xi^2}{4\rho}\right)}}{\Gamma\left(\sigma-\mu\right)}\int_{\xi}^{\infty}x\left(x^2-\xi^2\right)^{\sigma-\mu-1}\,e^{-\left(\frac{x^2}{4\rho}\right)}\,\phi_4\left(x,\xi\right)dx \tag{3.10}$$

Now, multiplying (3.7) by $\xi^{2(\mu+m)-1}$ **and applying the operator** $\left(\frac{d}{d\xi^2}\right)^m$**, we see in view of (2.6) that**

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)}P_{n+p,\mu}\left(\xi,-\rho\right)=H_2\left(\xi,-\rho\right),z<\xi<h \tag{3.11}$$

Where $H_2\left(\xi,\rho\right)=\xi^{1-2v}\left(\frac{d^2}{d\xi^2}\right)^m\left[\xi^{2(\mu+m)-1}\,F_3\left(\xi,\rho\right)\right]$ **(3.12)**

The left hand sides of (3.11), (3.5), (1.2), (1.4) are now identical and an application of the orthogonality relation (2.2) gives the solution of the quadruple series equations (1.1) – (1.4) in the form

$$A_n=\frac{\Gamma\left(\mu+\frac{1}{2}\right)}{2^{4(n+p)}\left(n+p\right)!}\left[\begin{array}{l}\int_0^y W_{n+p,\mu}\left(\xi,\rho\right)H_1\left(\xi,\rho\right)dR\left(\xi\right)+\\ \int_y^z W_{n+p,\mu}\left(\xi,\rho\right)\phi_2\left(\xi,\rho\right)dR\left(\xi\right)+\\ \int_z^h W_{n+p,\mu}\left(\xi,\rho\right)\phi_3\left(\xi,\rho\right)dR\left(\xi\right)+\\ \int_h^{\infty} W_{n+p,\mu}\left(\xi,\rho\right)H_2\left(\xi,\rho\right)dR\left(\xi\right)\end{array}\right] \tag{3.13}$$

Where H₁(ξ, ρ) and H₂(ξ, ρ) are the same as defined by (3.6) and (3.12) respectively and dR(ξ) is defined by (2.3).

Using the relation (2.1) and setting $B_n = A_n(-1)^{n+p} 2^{2(n+p)} (n+p)!$**, we find that the equations (1.1), (1.2), (1.3) and (1.4) transforms into**

$$\sum_{n=0}^{\infty} \frac{B_n \rho^n}{\Gamma\left(\nu+\frac{1}{2}+n+p\right)} L_{n+p}^{\left(\nu-\frac{1}{2}\right)}\left(\frac{x^2}{4t}\right) = t^{-\rho}\phi_1(x,t), 0<x<y \tag{3.14}$$

$$\sum_{n=0}^{\infty} \frac{B_n t^n}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)} L_{n+p}^{\left(\nu-\frac{1}{2}\right)}\left(\frac{x^2}{4t}\right) = t^{-\rho}\phi_2(x,t), y<x<z \tag{3.15}$$

$$\sum_{n=0}^{\infty} \frac{B_n \rho^n}{\Gamma\left(\nu+\frac{1}{2}+n+p\right)} L_{n+p}^{\left(\nu-\frac{1}{2}\right)}\left(\frac{x^2}{4t}\right) = t^{-\rho}\phi_3(x,t), z<x<h \tag{3.16}$$

$$\sum_{n=0}^{\infty} \frac{B_n t^n}{\Gamma\left(\mu+\frac{1}{2}+n+p\right)} L_{n+p}^{\left(\nu-\frac{1}{2}\right)}\left(\frac{x^2}{4t}\right) = t^{-\rho}\phi_4(x,t), h<x<\infty \tag{3.17}$$

Where $t \geq \rho > 0$ **and their solution is given by**

$$B_n = \frac{\Gamma\left(\mu+\frac{1}{2}\right)\Gamma(n+p+1)}{2^{\nu+\frac{1}{2}} \rho^{\nu+\frac{1}{2}+2n}} \left[\begin{array}{l} \int_0^y e^{-\left(\frac{\xi^2}{4\rho}\right)} L_{n+p}^{\left(\nu-\frac{1}{2}\right)}\left(\frac{\xi^2}{4\rho}\right) H_1(\xi,\rho)\,dR(\xi) \\ +\int_y^z e^{-\left(\frac{\xi^2}{4\rho}\right)} L_{n+p}^{\left(\nu-\frac{1}{2}\right)}\left(\frac{\xi^2}{4\rho}\right) \phi_2(\xi,\rho)\,dR(\xi) \\ +\int_z^h e^{-\left(\frac{\xi^2}{4\rho}\right)} L_{n+p}^{\left(\nu-\frac{1}{2}\right)}\left(\frac{\xi^2}{4\rho}\right) \phi_3(\xi,\rho)\,dR(\xi) \\ +\int_h^\infty e^{-\left(\frac{\xi^2}{4\rho}\right)} L_{n+p}^{\left(\nu-\frac{1}{2}\right)}\left(\frac{\xi^2}{4\rho}\right) H_2(\xi,\rho)\,dR(\xi) \end{array} \right] \tag{3.18}$$

The solution of quadruple series equations involving generalized Laguerre polynomials can be obtained independently by the above procedure.

REFERENCES

1. Erdelyi, A. etal. (1953), Higher Transcendental functions, Vol.II, Mc Graw Hill, New York.
2. Erdelyi, A. etal. (1954), Tables of integral transforms, Vol.II, Mc Graw Hill, New York.
3. Haimo, D.T. (1966), Expansions in terms of generalized heat polynomials and their appell transforms, J. Math. Mech; 15, 735-758.
4. Rosenbloom, P.C and Widder, D.V. (1959), Expansions in terms of heat polynomials and associated function, Trans. Amer. Math. Soc, 92, 220-266.
5. Lowndes, J.S. (1969), Some dual series equations involving Laguerre polynomials, Pacific J. Math, 25, 123-127.
6. Panda, R. (1977), Certain dual series equations involving Laguerre polynomials Nederl. Akad, Wetensh. Proc. Ser. A80 = Inday Math, 39, 122-127.
7. Srivastava, H.M. (1969), A note on certain dual series equations involving Laguerre polynomials, Pacific J. Math, 30, 525-527.
8. Narain,Kuldeep (2019) , Certain Quadruple Series Equations with Jacobi Polynomials as Kernels, International Journal of Mathematics Research; 11 , 55 -58.